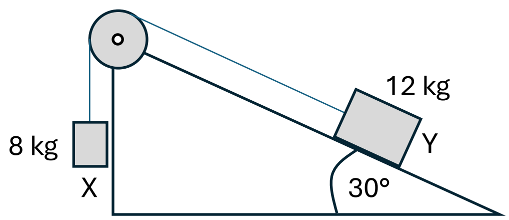
Teacher notes
Topic A

Three ways to solve a problem

A block X of mass 8 kg is connected with a string via a pulley to another block Y of mass 12 kg on an inclined plane making an angle of 30° to the horizontal.



X and Y are released from rest. Frictional and air resistance forces are negligible.

What is the total kinetic energy of X and Y after X falls a vertical distance of 5 m?

(Take $g = 10\ \text{m s}^{-2}$.)

Answer

Most would try to solve this by finding the speed of the blocks. This requires applying Newton's second law to each block to find the acceleration:

$80 - T = 8a$

$T - 120 \times \sin 30^{\circ} = 12a$

Adding we get

$20 = 20a \quad \Rightarrow \quad a = 1 \text{ m s}^{-2}$.

Hence $v^2 = 2as = 2 \times 1 \times 5 = 10 \text{ m}^2 \text{ s}^{-2}$.

The total kinetic energy is then $\frac{1}{2}mv^2 + \frac{1}{2}Mv^2 = \frac{1}{2} \times 8 \times 10 + \frac{1}{2} \times 12 \times 10 = 40 + 60 = 100 \text{ J}$.

But there are more elegant ways that take less time!

Applying the work kinetic energy relation, we know that

$$W_{\text{net}} = \Delta E_{\text{K}}$$

Now, $W_{\text{net}} = +mgs - mgs\sin 30^{\circ} = +8 \times 10 \times 5 - 12 \times 10 \times 5 \times \frac{1}{2} = 400 - 300 = 100 \text{ J}$.

This is the final kinetic energy since the initial is zero. Notice that we did not include the work of the normal force on Y because that work is zero; the force is at right angles to the displacement. We also did not include any work done by the tension force; the tension on Y does some work but the tension on X does the exact opposite so the total work by the tension is zero.

Another way involves the result:

$$W_{\text{ext}} = \Delta E_{\text{T}}$$

As discussed in a previous note, by external work we mean the work done by non-conservative forces. Here there are none and so $W_{\text{ext}} = 0$. Hence

$$0 = \Delta E_{\text{K}} + Mgs\sin 30^{\circ} - mgs$$

where ΔE_{K} is the change in total kinetic energy. Since the initial kinetic energy is zero this is also the final kinetic energy we seek. Hence

$\Delta E_{\text{K}} + 12 \times 10 \times 5 \times \frac{1}{2} - 8 \times 10 \times 5 = 0$

$\Delta E_{\text{K}} = 8 \times 10 \times 5 - 12 \times 10 \times 5 \times \frac{1}{2} = 400 - 300 = 100 \text{ J}$